Azonosító jel:

ÉRETTSÉGI VIZSGA • 2012. október 16.

MATEMATIKA
ANGOL NYELVEN

EMELT SZINTŰ
ÍRÁSBELI VIZSGA

2012. október 16. 8:00

Az írásbeli vizsga időtartama: 240 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to candidates

1. The time allowed for this examination paper is 240 minutes. When that time is over, you will have to stop working.

2. You may solve the problems in any order.

3. In Section II, you are only required to solve four out of the five problems**. When you have finished the examination, write in the square below the number of the problem NOT selected.** *If it is not clear* for the examiner which problem you do not want to be assessed, then problem 9 will not be assessed.

4. In solving the problems, you are allowed to use a calculator that cannot store and display verbal information. You are also allowed to use any book of four-digit data tables. The use of any other electronic device, or printed or written material is forbidden.

5. **Always write down the reasoning used in obtaining the answers, since a large part of the attainable points will be awarded for that.**

6. **Make sure that the calculations of intermediate results are also possible to follow.**

7. In solving the problems, theorems studied and given a name in class (e.g. the Pythagorean theorem or the altitude theorem) do not need to be stated precisely. It is enough to refer to them by the name, but their applicability needs to be briefly explained. Reference to other theorem(s) will only be awarded full mark if the theorem and all its conditions are stated correctly (proof is not required), and the applicability of the theorem to the given problem is explained.

8. Always state the final result (the answer to the question of the problem) in words, too.

9. Write in pen. Diagrams are also allowed to be drawn in pencil. If you cancel any solution or part of a solution by crossing it over, it will not be assessed.

10. Only one solution to each problem will be assessed. In the case of more than one attempt to solve a problem**, indicate clearly** which attempt you wish to be marked.

11. **Do not write anything in the grey rectangles.**

I.

1. In a new kind of lottery, there are 5 million tickets issued. One ticket costs 200 forints. Each ticket reads either PRIZE or NO PRIZE. A prize winning ticket also specifies the amount won by the ticket holder. The accompanying table shows the distribution of the prizes to be won with the 5 million tickets.

number of tickets	prize (forints)
4	10 000 000
40	50 000
800	10 000
150 000	1 000
400 000	500
1 000 000	200
3 449 156	0

a) Provided that all tickets are sold out and the winners claim all the prizes, what would be the difference of the income gained from selling the tickets and the total of the prizes paid to the winners?

b) What is the probability that the player buying the first ticket when the tickets are issued will win a prize that is more than the price paid for the ticket?

c) Calculate the expected value of the profit made by the player buying the first ticket. (In calculating the expectance of the profit, consider the price of the ticket as well as the prizes won.)

a)	3 points	
b)	4 points	
c)	4 points	
T.:	11 points	

2. The sum of two real numbers is 29. If 15 is subtracted from one number and 15 is added to the other number, the product of the two results obtained will be equal to five times the product of the original two numbers. What may be these two numbers?

T.:	13 points	

3. For each of the three expressions below, determine the largest subset of the set of real numbers on which the expression is meaningful.

a) $\cos\left(\log_2 \sqrt{x}\right)$

b) $\sqrt{\log_2(\cos x)}$

c) $\log_{\sqrt{x}}(\cos^2 x)$

a)	3 points	
b)	5 points	
c)	5 points	
T.:	13 points	

4. A sailing ship set out to circumnavigate the Earth has sunk at a distance of 16 km to the east of a small island in the Pacific Ocean. The crew are waiting for help in a lifeboat. They have a device for sending distress signals, but it only has a range of 6 km. When the sailing ship sank, there was an ocean liner to the south of the island, at a distance of 24 km from the island. The ocean liner is moving uniformly to the northeast while the crew of the sunken sailing ship are continually sending distress signals from the spot where their ship sank.

a) Prove that the crew of the ocean liner may receive the distress signals.

An aeroplane travelling at an altitude of 1.5 km is exactly over the island when its instruments detect the **ocean liner** that has covered a distance of 20 km since the sailing ship sank.

b) At what angle of depression is the liner detected by the aeroplane? Give your answer in degrees, rounded to the nearest whole degree.

In your calculations, ignore the curvature of the Earth.

a)	7 points	
b)	7 points	
T.:	14 points	

II

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

5. Given two parallel lines *e* and *f*, 5 distinct points are marked on *e* and 4 distinct points are marked on *f*.

a) How many lines (different from *e* and *f*) are determined by these 9 points?
How many triangles can be made, such that each vertex is taken from the 9 given points?
How many quadrilaterals can be made, such that each vertex is taken from the 9 given points?

b) Each of the 9 points is coloured at random, either blue or red. What is the probability that the 5 points on line *e* will all be identical in colour, and the 4 points on line *f* will also be identical in colour?

a)	11 points	
b)	5 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

6. In the International Contest of Robot Operated Electric Cars, the participating models are powered by rechargeable batteries. The car of the Hungarian competitors covers 45 km in the first hour. Owing to a decrease in battery power, the distance covered in the second hour is less than in the first hour, the distance in the third hour is less than in the second hour, and so on: the distance covered in the nth hour after starting out is always 95.5% of the distance covered in the $(n-1)$th hour ($n \in \mathbf{N}$ and $n > 1$).

a) What distance does the Hungarian car cover in the 10th hour?
Round your answer to the nearest kilometre.

In the Contest, there are races organized in various categories. The regulations of one of the categories allow the replacement of batteries during the race, too. The engineers of the Hungarian team have calculated that it is not worth replacing the battery yet in a particular hour if the distance covered in that hour is at least 20 km.

b) Which is the earliest hour after start when it may be worth replacing the battery?

In the "Death from Exhaustion" category, participants aim at covering the largest distance without battery recharge or replacement. The world record of 1100 km is held by the vehicle of a Japanese team.

c) Will the Hungarian car be able to beat the world record in the "Death from Exhaustion" category?

a)	4 points	
b)	6 points	
c)	6 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

7. A factory is planning to manufacture baking tins of 4000 cm^3 volume that have the shape of a rectangular prism with a square base, open at the top. The outer surface of the tins is to be coated with heat resistant enamel paint. (The inner surface will be coated with a different material.)

a) Calculate what the surface area to be coated with enamel paint would be if the height of the lateral faces of the tin were 6.4 cm.

b) The engineers of the factory designed the dimensions of the tins to minimize the amount of enamel paint needed. Calculate the length of the base edge of the tins manufactured.

c) Quality control statistics revealed that the probability of a tin selected at random being faulty is 0.02. What is the probability that there will be exactly 2 faulty tins in a shipment of 50 tins delivered to a store chain?

a)	3 points	
b)	9 points	
c)	4 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

8. The vertices of a triangle ABC in the coordinate plane are $A(2;1)$, $B(7;-4)$, $C(11;p)$. Determine the **exact value** of the parameter p if the interior angle at vertex B is 60º.

T.:	16 points	

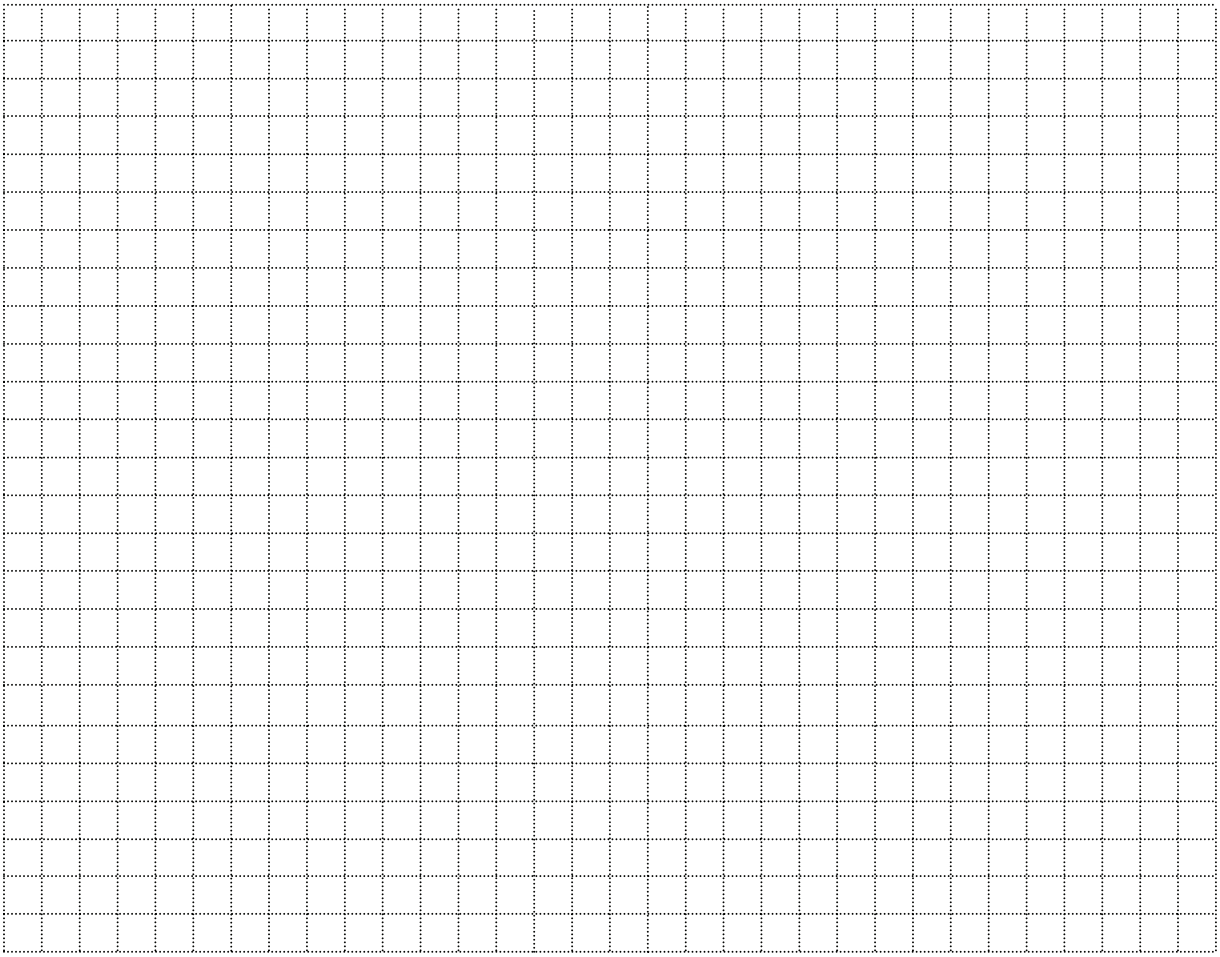
You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

9. **a)** Decide about each of the two statements below whether it is true or false.

(1) There exists a simple graph on 5 points that has 11 edges.
(2) If the degree of each vertex of a simple graph on 5 points is at least three, then it has a vertex of degree four.

b) The points A, B, C, D and E are the vertices of a complete graph on 5 points. Six edges are selected at random and coloured. What is the probability that the graph consisting of the points A, B, C, D, E and the coloured edges will not be connected?

a)	6 points	
b)	10 points	
T.:	16 points	



	number of problem	maximum score	points awarded	maximum score	points awarded
Part I	1.	11		**51**	
	2.	13			
	3.	13			
	4.	14			
Part II		16		**64**	
		16			
		16			
		16			
		← problem not selected			
Total score on written examination				**115**	

date examiner

	elért pontszám **egész számra** kerekítve / points awarded, rounded to **integer**	programba beírt **egész** pontszám / **integer** score entered in program
I. rész / Part I		
II. rész / Part II		

javító tanár / examiner jegyző / registrar

dátum / date dátum / date